# Kinematic-based Framework for Autonomous Ground Vehicle Trajectory Planning

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## Introduction/Motivation

- One of the major research topics in the area of Autonomous Vehicles is trajectory planning and tracking control problem
  - Safe Maneuver
  - Dynamically feasible
- Methods:
  - Kinematic based
    - Low speed
    - No unknown parameter
    - No slippage consideration
  - Dynamic based
    - High speed and harsh road
    - Unknown parameters identification
    - Consider slippage
- To avoid the estimation error and to use the simplicity of kinematic model at low-speed conditions, both kinematic and dynamic models are deployed in a cascade structure.
- It allows the local trajectory planning, and tracking control to be integrated. (see Figure 1)

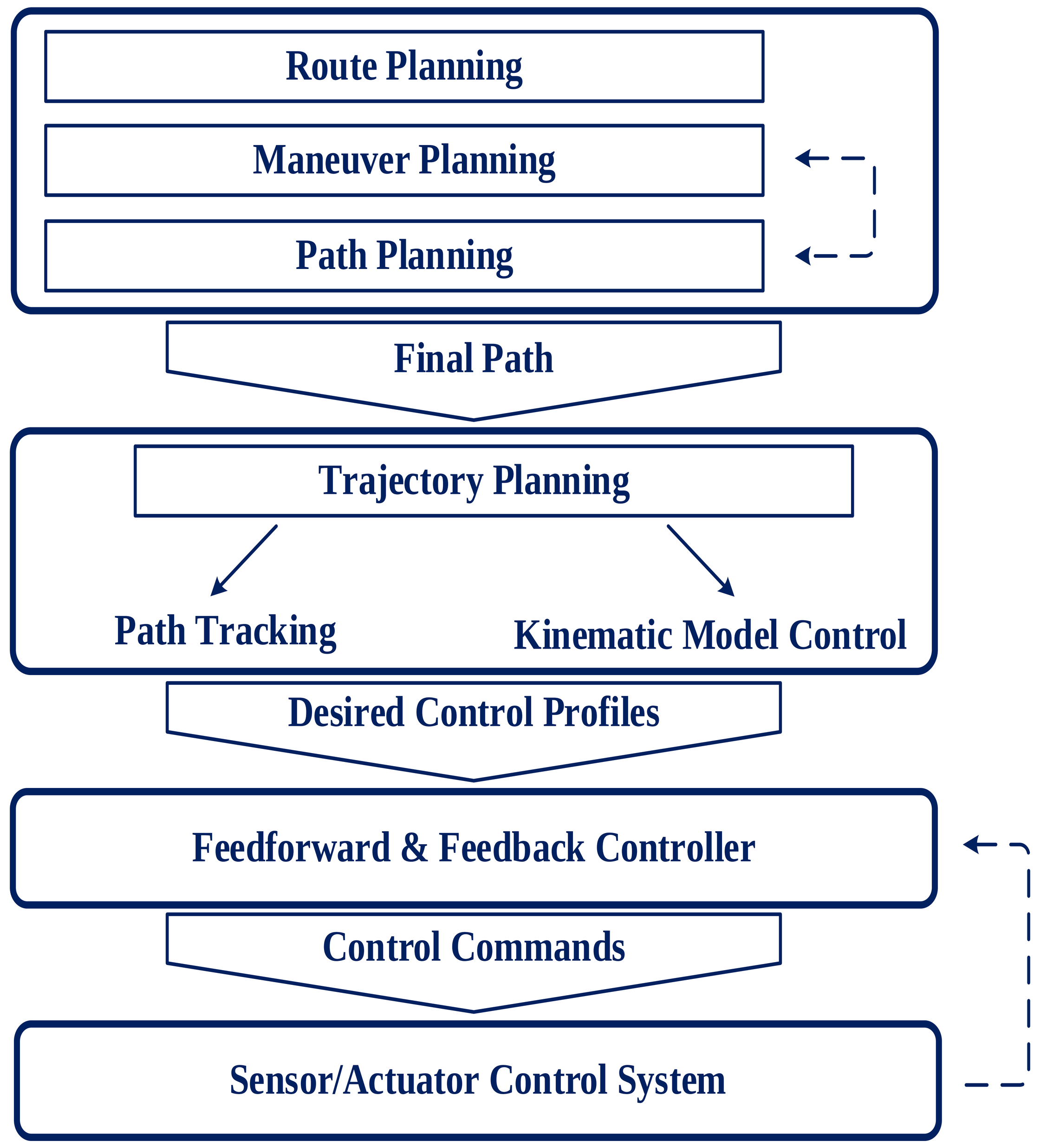


**Figure 1.** Cascade motion planning and control scheme

## Technical Challenges and Gaps

- In kinematic-dynamic cascade structures:
  - Linear model is employed for trajectory planning
  - Control inputs and trajectory are represented by specific geometric functions
- These studies fail to provide the optimal solution because of restricting the solution space.

## Objectives

- Proposing a novel cost function to minimize the error of position, velocity, and acceleration in the trajectory planning module on the second level of the cascade structure shown in Fig. 1.
- Not presuming the linearity of the trajectory planner and not restricting the input space and trajectory to any certain parametric class of functions, e.g. Bezier curves, splines, and polynomials

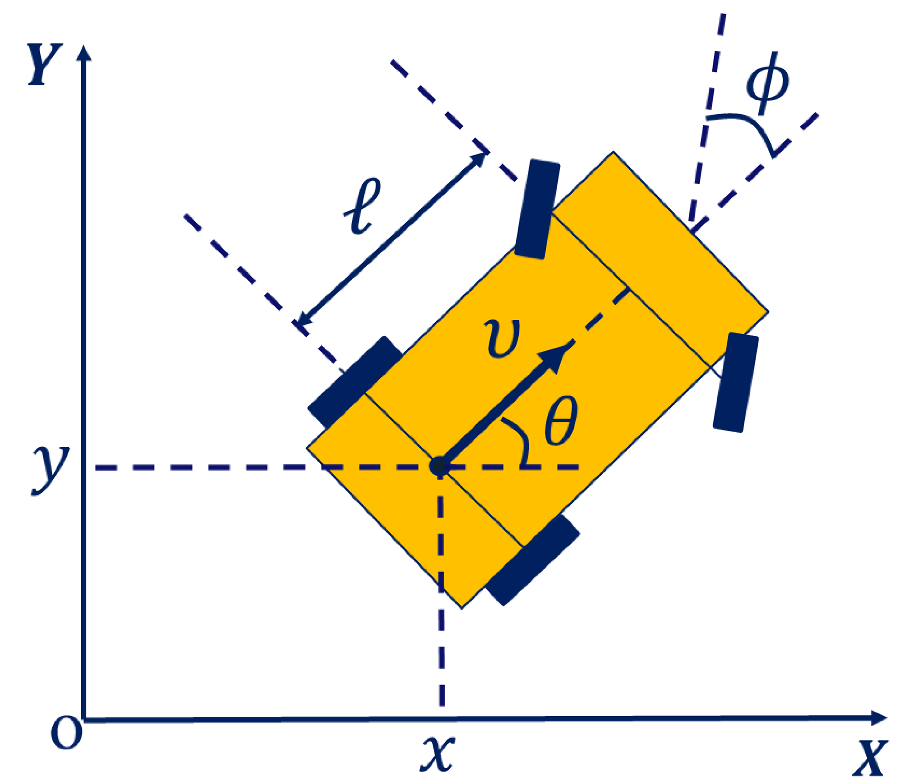


**Figure 2.** Rear-wheel car-like vehicle model

## problem Formulation

**A. Model**

Rear-wheel car like vehicle model is used in this work. The kinematics of the vehicle can be expressed as follows (Figure 2):

$$\dot{x}(t) = v(t)\cos(\theta(t))$$
$$\dot{y}(t) = v(t)\sin(\theta(t))$$
$$\dot{\theta}(t) = \frac{1}{\ell} v(t)\tan(\varphi(t))$$
$$\dot{v}(t) = a(t)$$

**B. Constraints**

- States

$$f_i(\boldsymbol{x}(t), t) \geq \boldsymbol{0} \quad i = 1,2,\dots,l$$

- Control inputs

$$\boldsymbol{u}_{min} \leq \boldsymbol{u}(t) \leq \boldsymbol{u}_{max}$$

**C. Cost Function**

$$\min J = \int_{t_0}^{t_f} \left(e_{position}^2 + e_{velocity}^2 + e_{acceleration}^2\right) dt$$

$$\text{Subject to } \begin{cases} \textit{Kinematic model} \\ \textit{State constraints} \\ \textit{Control constraints} \end{cases}$$

**D. Boundary Conditions**

Initial and Final points are fixed. Also, final time and reference path is given by path planner.

## Solution Method and Results

- The Variational approach is used to find the control variables by solving an optimal control problem, which leads to a set of two-point boundary value (TPBV) nonlinear differential equations.
- The optimal control variables and the trajectory are found by solving a set of nonlinear differential equations numerically using the collocation method.
- The performance of the proposed method is evaluated in multi-curve road scenario as shown in Figures 3- 5.

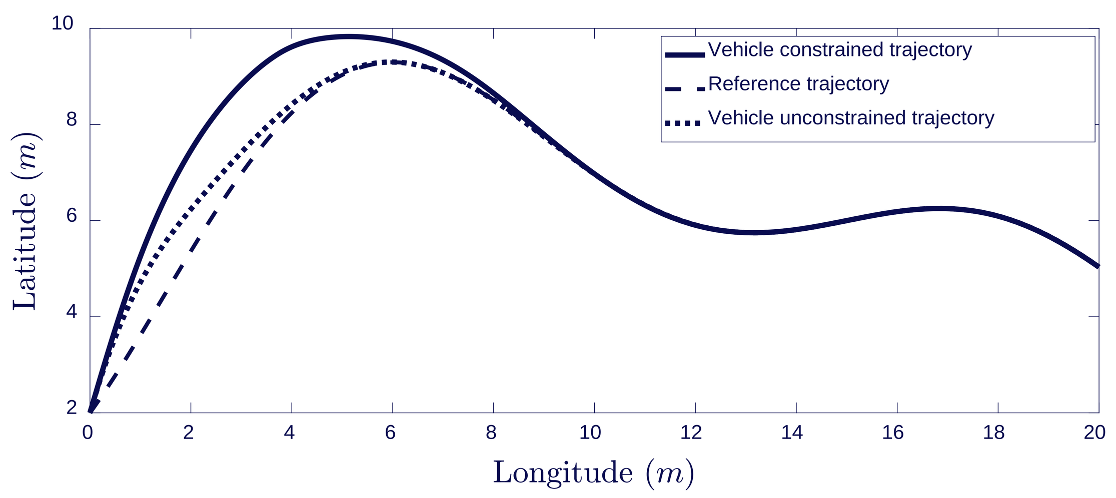


**Figure 3.** Vehicle trajectory

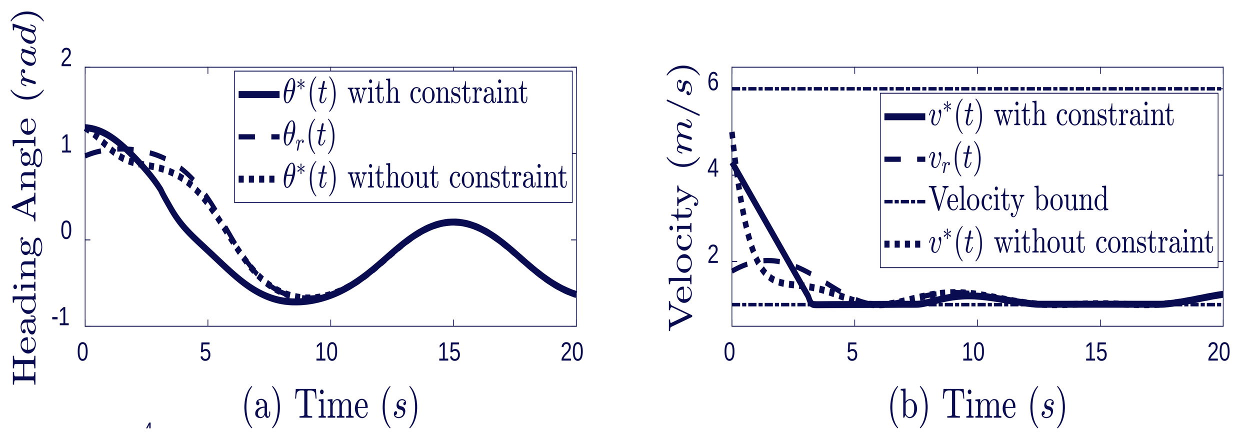


**Figure 4.** (a) heading angle, (b) velocity, and (c) augmented state

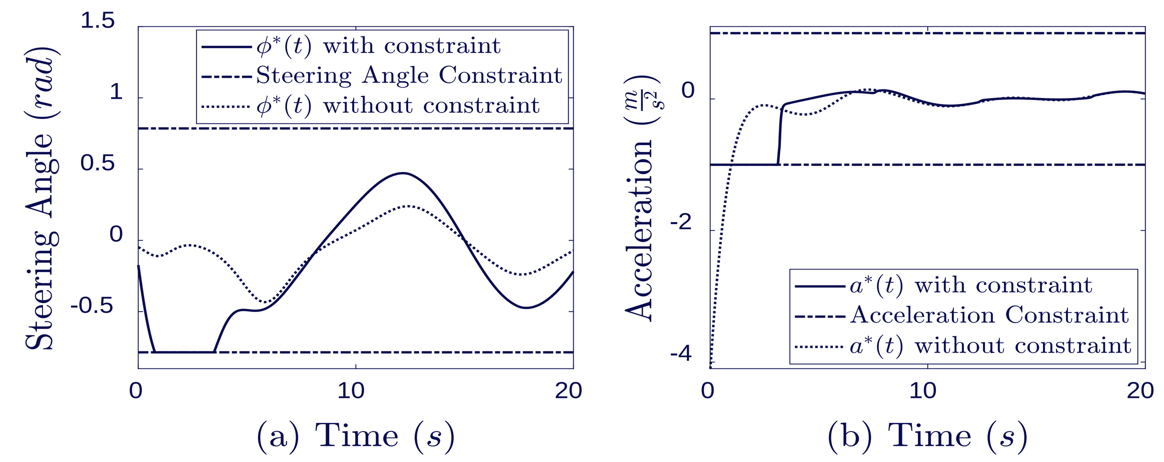


**Figure 5.** (a) steering angle, (b) acceleration

## Conclusion and Future Work

**In this work:**

- The non-linear vehicle kinematic model is considered in trajectory planning.
- Optimal vehicle trajectory and control variables are calculated in the same unit using the calculus of variation.
- No specific parametrized geometric representation is considered for the variables.

**In the future works:**

- Using the more complex model with more states (steering angle rate and jerk)
- Obstacle avoidance can be addressed by the extension of this framework.

## Publications

[1] Majd, K., Razeghi-Jahromi, M., & Homaifar, A. (2018). Optimal Kinematic-based Trajectory Planning and Tracking Control of Autonomous Ground Vehicle Using the Variational Approach. *Intelligent Vehicles (IV) Symposium*. Changshu, Suzhou, China: IEEE.

[2] Majd, K., Razeghi-Jahromi, M., Ramyar, S., & Homaifar, A. (2018). Model-based autonomous ground vehicle trajectory optimization and tracking using the Variational approach. *Conference on Decision and Control (CDC)*. Miami Beach, FL, USA: IEEE. (submitted).

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